Problem **17**

In this problem, we have a disk attached to a spring cake. Initially, we need to apply a moment of negative 40 Newton meters to maintain the system in equilibrium. At time t equals to zero, this moment is suddenly removed and the disk starts to oscillate. We're asked, with what period tau does the system oscillate, we can also assume the small angle approximation where sine of theta is approximately equal to theta, and cosine of theta is almost equal to one. So first, we start with a free body diagram of the system. So we start with our disk, a radius r, and we pin it about its center, which we're going to call Oh. And we are going to have reaction forces at the center, our x and our y. And we're going to have our force due to the spring, which is going to pull to the left in this, we're going to call Fs the force due to the spring. And this acts at point eight. So this up here would be point A, this down here would be point O. And the distance between point A and O is the vector R, A with respect to Oh, and we'll see why we need that later. But these are all the forces that are acting on this system, right? We're neglecting gravity, we're not taking into account gravity in this case. So there's two reaction forces, our x and our y, and the forest is spring. So what we need to do next is apply equilibrium, right. And again, this is not going to be static equilibrium, because we're going to have an i alpha or M A term, right? In this case, we're going to take the sum of moments about Oh, right. And this is going to be equal to i o alpha, we can also take the sum of forces in x and y. But that would not help us solve this problem. So let's take the sum of moments. So we know that there's only one, we're going to take the sum of moments about, oh, because we can eliminate the moments caused by these forces, right, because they act at this point. So we only have one force, that creates a moment, and that is the force due to that spring, right. And so what we're going to do now is just go ahead and calculate this moment. So the moment caused by that force, is going to be equal to r of a with respect to O cross F of s. And this is going to be equal to pi naught alpha, right? This is the vectorial equation. So alpha will be in the k hat direction force is in the negative x direction, and r is in the positive y, right? Because we define this as positive x is positive y, and this as a positive rotation in about Zed, right? So what we're going to do next is we're going to replace and complete this cross product, replace these variables here and do the cross product. So if we look at the diagram here, we know that the radius points in a positive y direction, so positive j, the force points in the negative x, so negative i hat direction. And if we cross product, we take a cross product of our cross F, we got something that points in the k hat direction, right? So that will be into or out of the page, depending on if it's positive, or if it's negative. And this matches with our alpha, right? Alpha, we said, we'll be either about will be a rotation about Zed, right? This is going to be alpha. Again, I haven't assigned a direction yet. We're going to find the direction with our signs. But alpha is about said, so it's going to be in the positive or negative k hat direction. So we can get rid of this cross product and multiply the R and F and just add the k hat direction, right? So we simplify that vectorial equation into a scalar equation. So let's get let's see what R is. Um, so r is just going to be the radius. And what is F? Well f is just equal to k x right? So f s is equal to k x, right, where x is the, how much we stretch or contract to the spring, depending on which way you're going right? Now, this x here, we can relate to the rotation, right? Because x here is specific to like the circumference, this little distance here, this would be the amount that we stretch, right. And this distance here is equal to theta times are in that gives us our distance there. And again, this is assuming small angle approximation. So this here becomes k, r, theta, right. And we also need to keep into account that this force counteracts. So we need to put a negative in front. So this is going to be negative k, r, theta. Okay. And the other thing is this alpha term here, we can replace with our theta double dot, right? Because alpha is the angular acceleration, which is the second derivative in time of the angle, right? It's the same definition. And so that we can actually have everything in terms of theta. So let's replace everything from this equation with the terms that we have just arrived. So we said that our will will just be R, right? So this is r times F of s, which is negative k r, theta. And all of this will be in the k hat direction, and we said is going to be equal to I naught times alpha, which we said is theta double dot, right, we still need to derive i naught, but I naught, we can just drive for a disc. So I naught is equal to one half, and r squared. And if we plug in the values for m and R, so 15 kilograms and one meter is one half times 15 kilograms times one meter squared. I naught is going to be equal to 7.5 kilograms, meters squared. Okay, so we now know i naught, we know K, we know r, because this becomes r squared, we can isolate and solve for theta, but we can't directly solve because it becomes a differential equation, right? So we have I not theta double dot plus k r squared theta is equal to zero. Right? Now, this is a differential equation, right? This is an ordinary differential equation. And so we could solve it. But in this case, the question doesn't ask us to solve for theta with respect to time, right? It just asks us, what is the time period of the oscillation, and you can extract this quantity directly from the differential equation. So if we take this equation, and we make it of this form, we can extract omega n from here, right. So if we remove anything from the first term, we ensure that this term is zero, because we have no forcing function, the term that remains over here, that is going to be our natural frequency squared, right? From this natural frequency, we can then derive the period right because period and frequency are related. So let's go ahead and do that we get the following equation theta double dot plus k r squared over i naught theta is equal to zero. So, omega n squared is equal to k r squared over i naught. And we can solve for this omega n squared is also going to be equal to two Pi over tau. And again all squared this is because omega n is equal to pi over tau right? My time period is related to the natural frequency. And what we can do is we can plug in the values. So in software tau, right, so tau is equal to two pi times square root of I naught, which is 7.5 kilograms meters squared divided by k r squared. Okay. And when we plug in these values, we get 1.81 seconds, and this is going to be equal to tau. And this is our final answer